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# Surface spin waves in a semi-infinite magnetic superlattice with a single-ion uniaxial anisotropy

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**Abstract.** Surface spin waves in a semi-infinite magnetic superlattice with a single-ion uniaxial anisotropy are investigated by use of the transfer-matrix method. The dispersion equations for surface spin waves are obtained. We find that not all of the structures of the superlattice can excite the surface spin waves and that the anisotropy term need not favour the excitation of surface spin waves, but that it will certainly influence the value of the energy of the excited surface spin waves.

## 1. Introduction

Superlattices of excellent quality can now be synthesized, following the recent advances in epitaxial growth techniques, especially molecular beam epitaxy (MBE). On the other hand, superlattices have unique physical features which may be very different from those of their component materials [1]. We can devise the superlattices we need with the aid of theoretical studies. These factors have aroused great interest in superlattice materials in recent years. As regards spin excitations, there have been extensive theoretical studies of magnetic–non-magnetic superlattices [2–5], while superlattices consisting of two [6–8] or more [9] different magnetic materials are becoming the focus of attention. Spin-wave spectra of superlattices have been investigated experimentally by means of light scattering [10, 11].

Experiments have shown there are very strong anisotropy fields in magnetic materials, and that these play an important role in determining magnetic properties, such as spin-wave energy, transition temperature and magnetization. There have been many papers on this subject [12–15]; comparatively less attention has been paid to magnetic superlattices that lack full translational symmetry [16] (mainly because of the existence of a surface or impurities). As we know, in addition to bulk spin-wave excitations, there are also surface modes whose amplitudes decay exponentially from the surface to the interior in semi-infinite systems. The purpose of the present paper is to investigate surface spin waves in a semi-infinite ferromagnetic–ferromagnetic superlattice with a single-ion uniaxial anisotropy.

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## 2. Theory

The superlattice under study here is formed from two different ferromagnetic layered structures stacked alternately—material 1 with $n_1$ layers in all and material 2 with $n_2$ layers in all. For simplicity, we restrict ourselves here to the case of simple cubic ferromagnetic structure with nearest-neighbour exchange interaction; the interfaces are assumed to be (001) planes having the same lattice constant $a_0$. Each material (1 or 2) is characterized by its Heisenberg exchange interaction ($J_1$ or $J_2$) and spin ($S_1$ or $S_2$). The constant describing the exchange interaction between interface atoms is $J$. For a semi-infinite system, the elementary unit cell is indicated by $l = 0, 1, 2, \ldots$ and the periodic distance $D = (n_1 + n_2)a_0$; it is obtained from the infinite system simply by equating the interface exchange constant $J$ to zero between $l = -1$ and $l = 0$. Without loss of generality, we assume that the surface layer is of material 1 (see figure 1).

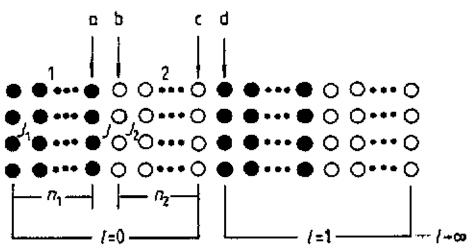


**Figure 1.** The geometry of the semi-infinite ferromagnetic–ferromagnetic superlattice model.

The Hamiltonians of materials 1, 2 and the interface are

$$
\begin{aligned}
\hat{H}_1 &= -\tfrac{1}{2}\sum_{(m,\,j)} J_1 \hat{S}_m \cdot \hat{S}_j - \sum_m D_1 (S_m^z)^2 \\
\hat{H}_2 &= -\tfrac{1}{2}\sum_{(m,\,j)} J_2 \hat{S}_m \cdot \hat{S}_j - \sum_m D_2 (S_m^z)^2 \\
\hat{H}_{\rm I} &= -\tfrac{1}{2}\sum_{(m,\,j)} J \hat{S}_m \cdot \hat{S}_j - \sum_l D_1 (S_l^z)^2 - \sum_k D_2 (S_k^z)^2
\end{aligned}
\tag{1}
$$

where $D_1$ and $D_2$ are single-ion uniaxial anisotropy parameters, which denote the strength of the anisotropy in materials 1 and 2 respectively. The sum over $l\,(k)$ in the interface Hamiltonian is taken over the interface atoms belonging to material 1 (2). All the sums are carried out between nearest-neighbour atoms.

The bulk spin-wave dispersion equations of materials 1 and 2 are given by

$$
\begin{aligned}
\hbar\omega &= 2D_1\langle S_1^z\rangle + 2J_1\langle S_1^z\rangle\left[3 - \cos(k_x a_0) - \cos(k_y a_0) - \cos(k_z a_0)\right] \\
\hbar\omega &= 2D_2\langle S_2^z\rangle + 2J_2\langle S_2^z\rangle\left[3 - \cos(k_x a_0) - \cos(k_y a_0) - \cos(k_z a_0)\right].
\end{aligned}
\tag{2}
$$

for low temperature ($T \ll T_{\rm c}$) the spins are fully ordered, so we have $\langle S_\alpha^z\rangle = S_\alpha\,(\alpha = 1, 2)$ [7].

The interface atomic planes a, b, c and d have the same nearest-neighbour environment as planes in the infinite system. If we write the spin-wave amplitudes of materials 1 and 2 as linear combinations of the positive- and negative-going solutions, we have

$$\hat{S}_m^\dagger = \{A_{l,j}\exp[\mathrm{i}\boldsymbol{k}_1\cdot(\boldsymbol{r}-\boldsymbol{r}_{l_1})] + B_{l,j}\exp[-\mathrm{i}\boldsymbol{k}_1\cdot(\boldsymbol{r}-\boldsymbol{r}_{l_1})]\}\exp(-\mathrm{i}\omega t) \tag{3}$$

for material 1 in the $l$th unit cell, and

$$\hat{S}_m^+ = \{C_{l,j}\exp[\mathrm{i}\boldsymbol{k}_2\cdot(\boldsymbol{r}-\boldsymbol{r}_{l_2})] + D_{l,j}\exp[-\mathrm{i}\boldsymbol{k}_2\cdot(\boldsymbol{r}-\boldsymbol{r}_{l_2})]\}\exp(-\mathrm{i}\omega t) \tag{4}$$

for material 2 in the $l$th unit cell, where $\boldsymbol{r}_{l_1} = [(l-1)D + a_0]\boldsymbol{i}_3$ and $\boldsymbol{r}_{l_2} = [(l-1)D + (n_1+1)a_0]\boldsymbol{i}_3$.

The equations of motion for spin waves in the interface atomic layers are obtained as follows:

$$\mathbf{A}\begin{pmatrix}\mathbf{A}_{l,j}\\ \mathbf{B}_{l,j}\end{pmatrix} = \mathbf{B}\begin{pmatrix}\mathbf{C}_{l,j}\\ \mathbf{D}_{l,j}\end{pmatrix} \qquad \bar{\mathbf{A}}\begin{pmatrix}\mathbf{A}_{l+1,j}\\ \mathbf{B}_{l+1,j}\end{pmatrix} = \bar{\mathbf{B}}\begin{pmatrix}\mathbf{C}_{l,j}\\ \mathbf{D}_{l,j}\end{pmatrix} \tag{5}$$

where the subscript $l$, $j$ indicates the $j$th atomic layer of the $l$th unit. We may rewrite equation (5) by use of the transfer matrix $\mathbf{M}$:

$$\begin{pmatrix}\mathbf{A}_{l+1,j}\\ \mathbf{B}_{l+1,j}\end{pmatrix} = \mathbf{M}\begin{pmatrix}\mathbf{A}_{l,j}\\ \mathbf{B}_{l,j}\end{pmatrix} \qquad \begin{pmatrix}\mathbf{A}_{l-1,j}\\ \mathbf{B}_{l-1,j}\end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix}\mathbf{A}_{l,j}\\ \mathbf{B}_{l,j}\end{pmatrix} \tag{6}$$

where $\mathbf{M} = \overline{\mathbf{A}}^{-1}\overline{\mathbf{B}}\mathbf{B}^{-1}\mathbf{A}$ with $\det(\mathbf{M}) = 1$ (its explicit form is given in appendix 1).

We know that Bloch's theorem does not hold in a semi-infinite superlattice, but we may use the following forms:

$$\begin{pmatrix}\mathbf{A}_{l+1,j}\\ \mathbf{B}_{l+1,j}\end{pmatrix} = \exp(-qD)\begin{pmatrix}\mathbf{A}_{l,j}\\ \mathbf{B}_{l,j}\end{pmatrix} \qquad \begin{pmatrix}\mathbf{A}_{l-1,j}\\ \mathbf{B}_{l-1,j}\end{pmatrix} = \exp(qD)\begin{pmatrix}\mathbf{A}_{l,j}\\ \mathbf{B}_{l,j}\end{pmatrix} \tag{7}$$

with the real part of $q$ positive; hence the amplitudes of surface modes decay exponentially as $l$ increases.

Combining (6) with (7), we get

$$\cos\mathrm{h}(qD) = \tfrac{1}{2}(M_{11} + M_{22}) \tag{8}$$

with $\mathrm{Re}(q) > 0$. This is the bulk spin-wave dispersion equation of the semi-infinite system.

Consider now the influence of the existence of a surface; the equation of motion for the surface layer is given by

$$\begin{pmatrix}A_{1,1}\\ B_{1,1}\end{pmatrix} = \mathbf{C}\begin{pmatrix}N_1\\ N_2\end{pmatrix} \tag{9}$$

where

$$N_1 = (\hbar\omega - 2D_{1\mathrm{s}}S_1 - 4J_{1\mathrm{s}}S_1 - J_1S_1 + J_{1\mathrm{s}}S_1\nu_1 + J_1S_1\exp(-\mathrm{i}k_1a_0))\exp(-\mathrm{i}k_1a_0)$$

$$N_2 = -(\hbar\omega - 2D_{1\mathrm{s}}S_1 - 4J_{1\mathrm{s}}S_1 - J_1S_1 + J_{1\mathrm{s}}S_1\nu_1 + J_1S_1\exp(\mathrm{i}k_1a_0))\exp(\mathrm{i}k_1a_0)$$

$$\nu_1 = 2(\cos(k_{1x}a_0) + \cos(k_{1y}a_0)) \qquad k_1 = k_{1z}$$

and **C** is a constant; here we have assumed that the surface exchange interaction is $J_{1s}$ and that the surface anisotropy parameter is $D_{1s}$.

Equations (6) and (7), applied for $l = 1$ and $j = 1$, combined with (9), then yield

$$\exp(-qD) = (N_2/N_1)M_{12} + M_{11} \tag{10}$$

$$N_1 N_2(M_{11} - M_{22}) - N_1^2 M_{21} + N_2^2 M_{12} = 0 \tag{11}$$

where we have employed the bulk spin-wave dispersion equation (8) and $\det(\mathbf{M}) = 1$.

Only those spin waves that can simultaneously meet the requirements of (10) and (11) and $\mathrm{Re}(q) > 0$ are true surface spin waves. In the next section, we discuss the influence of superlattice structures and anisotropy fields on surface spin waves.

## 3. Results

After some algebraic manipulation, the complex forms of (10) and (11) may be cast in the following simplified forms in the case where $k_1$ and $k_2$ are real (this case represents positive- and negative-going solutions that are of oscillatory form):

$$\exp(-qD) = (KE - LF + G)/A - [(KF + EL + H)/A]\mathrm{i} \tag{12}$$

$$LE - KF - H = 0 \tag{13}$$

with $\mathrm{Re}(q) > 0$, where $k_1$ and $k_2$ are determined by (2) for given $\hbar\omega$, $k_{\alpha x}$, $k_{\alpha y}$ ($\alpha = 1, 2$) and where the explicit representations of the real variables $A$, $K$, $E$, $L$, $F$, $G$, $H$ are given in appendix 2.

As in the above discussion, the necessary condition for the excitation of surface spin waves (SSW) is $\mathrm{Re}(q) > 0$; here $q$ is the wave-vector component describing the modulation of the envelope function from layer to layer. Equation (12) is an implicit dispersion relation for SSW. Our interest lies in the dependences of the geometrical structures and the anisotropy of the magnetic superlattice on the SSW; knowledge of these should enable us conveniently to tailor the desired superlattice.

In order to discuss the influence of superlattice structures and anisotropy strength on SSW, we take the case of $k_{\alpha x} = k_{\alpha y} = 0\,(\alpha = 1, 2)$, $n_2 = 20$, $\Delta = S_2/S_1 = \frac{2}{3}$, $D_{1s}/D_1 = 2.0$, $D_2/D_1 = 1.5$, $J_{1s}/J_1 = 2.0$, $J_2/J_1 = 2.0$ and $J/J_1 = \pm 1.5$ (positive and negative values of the interface exchange interaction constant $J$ represent ferromagnetic and antiferromagnetic exchange interaction between interface atomic layers).

Figure 2 shows the relations between the SSW energy and the superlattice structures—represented by the value of $n_1$ (with $n_2$ fixed)—where the interface atomic layers are subject to ferromagnetic exchange interaction in the case of: (*a*) no anisotropy field, $D_1/J_1 = 0.0$; (*b*) weak anisotropy fields, $D_1/J_1 = 1.0$; and (*c*) strong anisotropy fields, $D_1/J_1 = 10.0$ (the blanks represent the structures that cannot excite SSW).

We find, in a chosen material 2 with $n_2 = 20$ layers, that not all of the superlattice structures lead to the excitation of SSW. In some structures there is no SSW to be excited. In [9] it was pointed out that in superlattices with ferromagnetic or antiferromagnetic layers of thickness $d_1$ and non-magnetic layers of thickness $d_2$, SSW can be excited only if $d_1 > d_2$ in the magnetostatic limit, so that the amplitudes of SSW

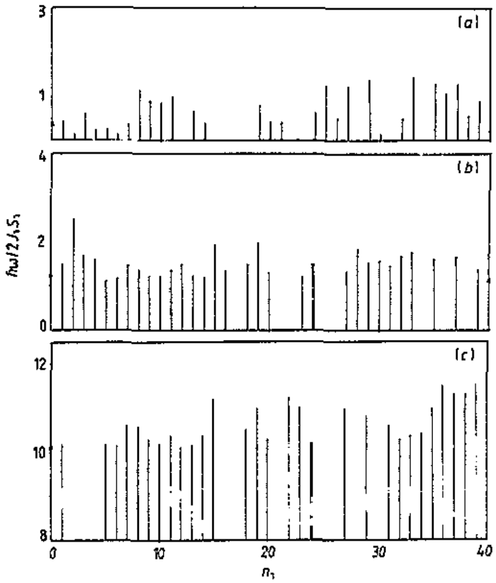


**Figure 2.** SSW energy versus superlattice structures—i.e. the value of $n_1$ ($n_2$ held constant)—for the ferromagnetic–ferromagnetic superlattice with ferromagnetic interface exchange interaction. We choose: (*a*) no anisotropy, $D_1/J_1 = 0.0$; (*b*) weak anisotropy, $D_1/J_1 = 1.0$; (*c*) strong anisotropy, $D_1/J_1 = 10.0$. The blanks represent structures that cannot excite SSW.

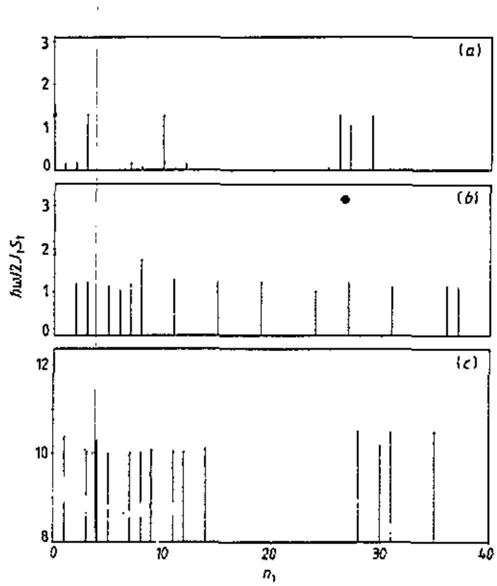


**Figure 3.** As figure 2, but with antiferromagnetic interface exchange interaction.

decay exponentially, as they should. In our ferromagnetic–ferromagnetic exchange model superlattice, the true SSW should satisfy this same condition, which requires that $n_1$ and $n_2$ must be matched.

Because of the existence of anisotropy fields, some superlattice structures that cannot excite SSW for the case without anisotropy fields may excite SSW, while some structures that can excite SSW in the absence of anisotropy fields may no longer be able to do so. The reason for this is that the anisotropy fields influence the SSW properties of exponential decay in the structures. In addition, it is natural, in the case of the superlattice consisting of only one component material, material 2 ($n_1 = 0$), that there are SSW that can be excited no matter whether there are anisotropy fields or not and no matter how strong the anisotropy fields are.

From the excited SSW energy point of view, apart from the existence of fluctuation between adjacent $n_1$ layers, it is obvious that the SSW energy increases as the strength of anisotropy field increases. This shows that the existence of anisotropy fields leads to the increase of excited SSW energy.

The case where interface atomic layers are subject to antiferromagnetic exchange interaction is illustrated in figure 3 (the other parameters are the same as in figure 2). We also find that not all of the possible structures of $n_1$ layers of material 1 can excite SSW (with $n_2$ fixed). Compared with the case shown in figure 2, the structures that can excite SSW have decreased remarkably in number, and even in the presence of anisotropy fields ($D_1/J_1 = 1.0$ and $D_1/J_1 = 10.0$) no SSW can be excited for $n_1 = 0$. This is exactly opposite to what is found in the ferromagnetic interface exchange interaction case. Although the excited SSW energy increases with the strength of the anisotropy fields, the energy and its fluctuation between adjacent $n_1$ layers decrease, correspondingly, from the values shown in figure 2. This can be understood by appreciating that the antiferromagnetic exchange interaction between interface atomic layers will hinder the excitation of SSW in our ferromagnetic–ferromagnetic superlattice, and so the number of structures, and the energy and fluctuation of the SSW will accordingly decrease.

In summary, we have studied the surface spin waves in superlattices consisting of two different ferromagnetic materials with a single-ion uniaxial anisotropy and with either ferromagnetic or antiferromagnetic interface exchange interaction. We find that not all of the arbitrary superlattice structures can excite surface spin waves. On comparing the ferromagnetic interface exchange interaction case with the antiferromagnetic case, the number of structures that can excite SSW, the SSW energy and the energy fluctuation between adjacent structures are found to increase strikingly. Single-ion uniaxial anisotropy need not favour the excitation of SSW, but its existence will lead to a requirement for more energy to excite SSW. What we have presented here is only a tentative study; it is our hope that it will promote research—either theoretical or experimental—on this subject in the future.

## Acknowledgments

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## Appendix 1.

The explicit form of the transfer matrix $\mathbf{M}$ is

$$\mathbf{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

where

$$M_{11} = \overline{g}_1 h_1[(\lambda_1\lambda_2 - J^2)^2 h_2\overline{g}_2 - (\lambda_1\overline{\lambda}_2 - J^2)^2\overline{h}_2 g_2]/A$$
$$M_{21} = \overline{g}_1 h_1[(\overline{\lambda}_1\overline{\lambda}_2 - J^2)(\lambda_1\overline{\lambda}_2 - J^2)\overline{h}_2 g_2 - (\overline{\lambda}_1\lambda_2 - J^2)(\lambda_1\lambda_2 - J^2)h_2\overline{g}_2]/A$$
$$M_{12} = g_1\overline{h}_1[(\lambda_1\lambda_2 - J^2)(\overline{\lambda}_1\lambda_2 - J^2)h_2\overline{g}_2 - (\lambda_1\overline{\lambda}_2 - J^2)(\overline{\lambda}_1\overline{\lambda}_2 - J^2)\overline{h}_2 g_2]/A$$
$$M_{22} = g_1\overline{h}_1[(\overline{\lambda}_1\overline{\lambda}_2 - J^2)^2 g_2\overline{h}_2 - (\overline{\lambda}_1\lambda_2 - J^2)^2\overline{g}_2 h_2]/A \qquad \text{(A1.1)}$$

with

$$g_\alpha = \exp(\mathrm{i}k_\alpha a_0) \qquad \overline{g}_\alpha = 1/g_\alpha = \exp(-\mathrm{i}k_\alpha a_0) \qquad (\alpha = 1, 2)$$
$$h_\alpha = \exp(\mathrm{i}n_\alpha k_\alpha a_0) \qquad \overline{h}_\alpha = 1/h_\alpha = \exp(-\mathrm{i}n_\alpha k_\alpha a_0) \qquad (\alpha = 1, 2)$$
$$\lambda_1 = J\Delta - J_1(\nu_1 + g_1) - (\hbar\omega/S_1 - 2D_1) + (Z-1)J_1$$
$$\lambda_2 = J/\Delta - J_2(\nu_2 + \overline{g}_2) - (\hbar\omega/S_2 - 2D_2) + (Z-1)J_2. \qquad \text{(A1.2)}$$

($\overline{\lambda}_\alpha$ has the same form as $\lambda_\alpha$ but with $\overline{g}_\alpha$ replaced by $g_\alpha$ $(\alpha = 1, 2)$)

$$\nu_\alpha = 2\cos(k_{\alpha x}a_0) + 2\cos(k_{\alpha y}a_0) \qquad (\alpha = 1, 2)$$

$$\Delta = S_2/S_1 \qquad Z = 6$$

$$k_\alpha = k_{\alpha z} \qquad (\alpha = 1, 2).$$

**Appendix 2.**

The real variables $A$, $K$, $E$, $L$, $F$, $G$, $H$ in (12) and (13) are

$$\begin{aligned}
A &= -4J^2 J_1 J_2 \sin(k_1 a_0)\sin(k_2 a_0) \\
K &= -\{[a^2 - J_1^2 S_1^2 \sin^2(k_1 a_0)]\cos(2k_1 a_0) - 2aJ_1 S_1 \sin(k_1 a_0)\sin(2k_1 a_0)\}/Q \\
E &= -4J_1^2 J_2^2[\cos(k_1\alpha_1)\sin(k_1 a_0) + r_1\sin(k_1\alpha_1)]r_1 r_4 - 4JJ_1J_2\{\sin(k_1\alpha_1)r_1 \\
&\quad \times [\sin(k_2\alpha_2) - \sin(k_2 d_2) + \sin(k_2(n_2+1)a_0)] \\
&\quad \times \Delta J_2 - 2J_1\sin(k_1\alpha_1)r_1 r_4/\Delta\} + J^2\{2\Delta^2 J_2^2 \sin(k_1\alpha_1) \\
&\quad \times [\sin(k_2\alpha_2) - \sin(k_2 d_2) + \sin(k_2(n_2+1)a_0)] \\
&\quad - 4J_1 J_2 \sin(k_1\alpha_1) r_1 r_4 + 4J_1^2 \sin(k_1\alpha_1) r_1 \sin(k_2\alpha_2)/\Delta^2\} \\
L &= \{[a^2 - J_1^2 S_1^2 \sin^2(k_1 a_0)]\sin(2k_1 a_0) + 2aJ_1 S_1 \cos(2k_1 a_0)\}/Q \\
F &= -4J_1^2 J_2^2[r_1\cos(k_1\alpha_1) - \sin(k_1 a_0)\sin(k_1\alpha_1)]r_1 r_4 - 4JJ_1J_2\{\cos(k_1\alpha_1)r_1 \\
&\quad \times [\sin(k_2\alpha_2) - \sin(k_2 d_2) + \sin(k_2(n_2+1)a_0)] \qquad \text{(A2.1)} \\
&\quad \times \Delta J_2 - 2J_1\cos(k_1\alpha_1)r_1 r_4/\Delta\} + J^2\{2\Delta^2 J_2^2 \cos(k_1\alpha_1) \\
&\quad \times [\sin(k_2\alpha_2) - \sin(k_2 d_2) + \sin(k_2(n_2+1)a_0)] \\
&\quad - 4r_1 r_4 \cos(k_1\alpha_1) + 4\cos(k_1\alpha_1)\sin(k_2\alpha_2)r_1/\Delta^2\} \\
G &= -8J_1^2 J_2^2 r_1 r_2 \sin(k_1 d_1)\sin(k_2 d_2) + 8JJ_1J_2[J_1 r_1 r_4 \sin(k_1 d_1)/\Delta \\
&\quad + \Delta J_2 r_2 r_3 \sin(k_2 d_2)] + 4J^2[J_1^2 r_1 \sin(k_1 d_1)\sin(k_2\alpha_2)/\Delta^2 + \Delta^2 J_2^2 r_2 \\
&\quad \times \sin(k_1\alpha_1)\sin(k_2 d_2) - J_1 J_2 r_3 r_4] \\
H &= 8J_1^2 J_2^2 r_1 r_2 \cos(k_1 d_1)\sin(k_2 d_2) - 8JJ_1J_2[J_1 r_1 r_6 \sin(k_1 d_1)/\Delta \\
&\quad + \Delta J_2 r_2 r_5 \sin(k_2 d_2)] - 4J^2[J_1^2 r_1 \cos(k_1 d_1)\sin(k_2\alpha_2)/\Delta^2 \\
&\quad + \Delta^2 J_2^2 r_2 \sin(k_2 d_2)\cos(k_1\alpha_1) - J_1 J_2 r_4 r_5]
\end{aligned}$$

where

$$
\begin{aligned}
&Q = a^2 + J_1^2 S_1^2 \sin^2(k_1 a_0) \\
&a = 2(D_1 - D_{1s})S_1 + (J_{1s} - J_1)\nu_1 S_1 + 4(J_1 - J_{1s})S_1 + J_1 S_1[1 - \cos(k_1 a_0)] \\
&r_i = 1 - \cos(k_i a_0) \qquad (i = 1, 2) \\
&r_{i+2} = \sin(k_i d_i) - \sin(k_i \alpha_i) \qquad (i = 1, 2) \\
&r_{i+4} = \cos(k_i d_i) - \cos(k_i \alpha_i) \qquad (i = 1, 2) \\
&d_i = n_i a_0 \qquad (i = 1, 2) \\
&\alpha_i = (n_i - 1) a_0 \qquad (i = 1, 2).
\end{aligned}
\tag{A2.2}
$$

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